Progression of Outcomes for Addition and Subtraction

<table>
<tr><th>Grade:</th><th>Kindergarten</th><th>Grade 1</th><th>Grade 2</th><th>Grade 3</th><th>Grade 4</th><th>Grade 5</th><th>Grade 6</th><th>Grade 7</th><th>Grade 8</th><th>Grade 9</th></tr>
<tr><td>Outcome</td><td>NK.2 Subitize numbers to 5.
NK.4 Partition whole numbers (0 to 10) into parts.
NK.5 Compare quantities using one-to-one correspondence.</td><td>N1.5 Compare sets of up to 20 elements to solve problems.
N1.8 Find a number that is 1 more, 2 more, 1 less and 2 less than a number up to 20
N1.9 Add and subtract numbers with answers to 20
N1.10 Addition and Subtraction Strategies
-count on
-count back
-make a ten
-use doubles
-use addition to subtract
P1.4 Record equalities using the equal symbol.</td><td>N2.2 Add and subtract numbers with answers to 100
-solve problems
-estimate sums and differences
P2.3 Demonstrate understanding of equality and inequality concretely and pictorially.</td><td>N3.2 Add and subtract whole numbers with answers to 1000
-estimate
-solve problems</td><td>N4.2 Add and subtract whole numbers with answers to 10 000
-estimate
-solve problems
N4.8 Add and subtract decimals to hundredths:
-compatible numbers
-mental math
-solving problems</td><td>N5.4 Computation using estimation
-front-end rounding
-compensation
-compatible numbers
-rounding
N5.7 Add and subtract decimals to thousandths</td><td>N6.3 Order of operations on whole numbers</td><td>N7.2 Add and subtract decimals to greater number of decimal places.
Use order of operations.
N7.5 add and subtract positive fractions and mixed numbers with like and unlike denominators concretely, pictorially and symbolically
N7.6 add and subtract integers concretely, pictorially and symbolically</td><td></td><td>N9.2 Compare, order, add and subtract rational numbers (in problem solving format)</td></tr>
<tr><td rowspan="3">Progression of Learning</td><td colspan="9">Addition and Subtraction (all in connection with each other, in equal weight, not taught in isolation or in sequence)
Counting; knowing number names, knowing counting principles, skip counting, counting on, counting all, counting back
Subitizing; building parts up to get a whole
Partitioning; breaking a whole into parts, joining, separating, part-part-whole, flexible with numbers
Strategies; one more, one less, five frame, ten frame, doubles, double plus one, ten facts, make ten (up through ten, back through ten), flexible with numbers (number sense), Students who only memorize facts without understanding will have limited number sense
Relationship between addition and subtraction, inverse relationship between addition and subtraction
Properties; commutative, zero property, associative,
Equality; part-part-whole, partitioning
The Structure for Addition and Subtraction Problems, Developing Meaning for the Operation of Addition and Subtraction; join, separate, part-part-whole, compare</td></tr>
<tr><td colspan="9">Counting; Can the student say the number names in order to 10, to 20, to 100?
Can the student associate the numeral to the number name?
Does the student understand that number represents a quantity? Can they connect 5 to five objects?
Does the student know the counting principles?
Does the student know the skip counting sequence? Can (s)he state the number names in sequence? (i.e. 2, 4, 6, 8, 10, ….)
Can the student relate the skip counting sequence to groups of objects? (i.e. touch 2 items when saying two)
Does the student count a collection by counting all of the objects?
Does the student count a collection by counting on? (i.e. cover an initial pile of objects and then count on)
Does the student count back to determine how many are left? (i.e. take two objects away from a set of 7, now there are 7,6,5 objects)</td></tr>
<tr><td colspan="9">Subitizing; Does the student understand that number represents a quantity? (i.e. Can you tell me how many you see without counting?)
Use dot cards, use dice, use fingers (vary what you are using)</td></tr>
</table>

		How do you see it? (video by Jo Boaler conceptual learning 5.5 number sense) Does the student subitize larger quantities by breaking them into parts or does the student revert back to counting all?
		Partitioning; Can the student find any missing part of a part-part-whole relationship given concretely, pictorially and symbolically? (give examples) Can the student find any missing part of a part-part-whole relationship given in a story problem context? (give examples) Can the student represent and solve problems that represent the actions of joining, separating, comparing, and working with part-part-whole? (i.e. problems vary based on the result unknown, change unknown, initial unknown) Can the student create problems to represent the actions of joining, separating, comparing and working with part-part-whole? Can the student find multiple decompositions of any given number? (i.e. 2+3, 4+1, 2+2+1, …) Can the student use five as an anchor when decomposing numbers? Then 10, then 25……(i.e. 7 and 4 is equal to 7+3 and one more = 11)
		Basic Facts Strategies; (concretely, pictorially, symbolically) Can the student explain **how** they know the answer to 4+1? 9+8? One more 8+1= One less 6-1= Five frame 7 is the same as 5+2 5 facts 0+5, 1+4, 2+3 Ten frame 12 is the same as 10+2 10 facts 0+10, 1+9, 2+8, 3+7, 4+6, 5+5 Make ten (up through ten or up over ten, back through ten or back down through ten), for 8+5, think 8+2+3 which is the same as 10+3=13; or for 9+4 is the same as 9+1+3 which is the same as 10+3=13 Doubles 1+1, 2+2, 3+3, …. Doubles plus one 6+7 is the same as 6+6+1 Doubles minus one 5+6 is the same as 6+6-1 Flexible with numbers (number sense), being able to decompose numbers into friendly numbers for that question and then mentally adding (subtracting) the numbers to find the sum, or subtracting the numbers to find the difference. Students who only memorize facts without understanding how to decompose numbers may have limited number sense since they have not had time to play with breaking numbers apart and building them up again different ways. Think addition to subtract, some people prefer to think of adding up to subtract 12-7, think 7+?=12, 7+3=10, 10+2=12, so 3+2 is 5, therefore 12-7=5
		Addition and Subtraction Strategies; (for larger numbers) Extending the 10 facts 12+8=20, 34+6=40, 56+24=80 Extending Make ten (up through ten or up over ten, back through ten or back down through ten), for 18+5, think 18+2+3 which is the same as 20+3=23; or for 29+34 is the same as 29+1+33 which is the same as 30+33=63 Decomposing numbers different ways to add and subtract: 77+27 is the same as 75+2+25+2, or 100+4=104; Adding left to right Addition and subtraction using traditional algorithms (borrow/carry) NOTE: The empty number line is a visual tool to show students their thinking when using any of the above strategies or it can also used to show students how to add and subtract in parts.
		Addition and Subtraction with Decimals, Fractions, Integers, Percents
		Relationship; Does the student see the connection between the number families? Is the student able to find the whole by adding or subtracting? Does the student think addition for subtraction? Does the student see how addition and subtraction can ‘undo’ each other?

	Properties; concretely, pictorially, symbolically Does the student see that adding one creates the next counting number? Does the student see that subtracting one create the counting number that comes before? Can the student model adding or subtracting zero? Does the student see that 2+3 is the same as 3+2? (commutative property) Can the student regroup addends for ease of adding? (4+8+2+9+6 is more easily added as 4+6+8+2+9) (associative property)
	Equality; Does the student understand that the equal sign represents a balance (the fulcrum on a balance beam) with both sides representing the same quantity? a+b = c+d
Big Ideas	Addition can be thought of as physically or conceptually placing two or more quantities together. Subtraction can be thought of as taking an amount away from a given quantity, comparing two quantities, or finding a missing part given the whole and the other part. Students need to understand all three meanings of subtraction and this is best done through word story problems. Models can be used to solve contextual problems for all operations and to figure out what operation is involved in a problem regardless of the size of the numbers. Models can be used to give meaning to number sentences. Addition and subtraction are connected: addition names the whole in terms of the parts, and subtraction names the missing part. Addition and subtraction undo each other so therefore they are related inverse operations. Flexible methods of computation involve taking apart (decomposing) and combining (composing) numbers in a variety of ways. Flexible methods of computation requires a deep understanding of the operations and the properties of the operations (commutative, associative) Invented strategies provide flexible methods of computing that vary with numbers and the situation. Multi-digit numbers can be built up or taken apart in a variety of ways. Nearly all computational estimations involve using easier to handle parts of numbers or substituting difficult to handle numbers with close compatible numbers so that the resulting computations can be done mentally. Estimates are useful for both checking calculations and because sometimes an estimate is all that is needed.
Instructional Strategies	Have students add and subtract horizontally as often as they add and subtract vertically. Use the empty number line to model our own thinking when we add and subtract in parts. Allow students time to understand the different meanings for addition and subtraction before beginning to focus on mental math strategies for basic facts. Do this within the contexts of story problems. Allow students time to add and subtract using a variety of strategies and understanding what they are doing through the use of contextual problems and a variety of models before you introduce the traditional algorithm. Teach and work with addition and subtraction daily (through a number talk or calendar time) throughout the year to allow students time to become fluent with a variety of strategies. Games provide an opportunity for students to practice strategies in a relaxed, fun setting. Take time to show different ways/strategies for one expression (45+56). Take time to work with numbers in contextual situations, highlighting a variety of strategies to find a solution for a single problem.
Common Misconceptions	Students are introduced to one meaning for subtraction (take away), therefore students can only subtract by taking away or counting backwards. *Solution: students need to become familiar with all three meanings for subtraction through story problems.* Students may not move beyond counting on or counting back to add and subtract. They become fast counters and do not have to learn strategies, however once the numbers become large, counting is no longer an efficient or accurate strategy. *Solution: students need to have time to become familiar and fluent with a variety of strategies beyond counting.* Students are unable to solve contextual problems. If they only look at the key words (altogether, in all) to determine the operation, they will not be able to interpret different math problems they will encounter. *Solution: students need to solve many story problems involving a variety of contexts.* Students can follow one procedure but cannot explain what they are doing or transfer this knowledge to a new situation. *Solution: don't rush to teaching the traditional algorithms too soon, they are digit oriented and may interfere previous knowledge of number sense.*